

國立彰化高級中學 111 學年度資優班甄選—複選實作評量【數學科】答案卷

※答案須化成「最簡分數」或「最簡根式」。

一、填充題【第 1~4 題，每題 6 分；第 5~12 題，每題 7 分，共 80 分】

1.	2.	3.	4.
$\frac{401}{4}\pi$	-3	$-\frac{40400}{337}$	$-\frac{71}{72}$
5.	6.	7.	8.
$\frac{580}{21}$	48	1080000	27
9.	10.	11.	12.
$\frac{\sqrt{2}}{2}$	6000	$3\sqrt{7}$	73

二、計算證明題【每題 10 分，共 20 分。請寫出計算過程，否則不予計分】

1.

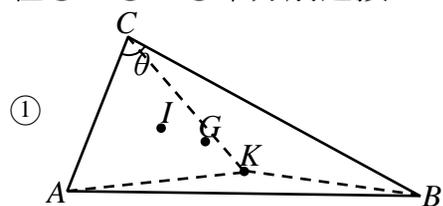
$$\sqrt{x^2+1} = |2x+1|+1 \Rightarrow x^2+1 = (|2x+1|+1)^2$$

$$\textcircled{1} \quad x \geq -\frac{1}{2} \text{ 時, } x^2+1 = (2x+2)^2 \Rightarrow 3x^2+8x+3=0 \Rightarrow x = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-4 \pm \sqrt{7}}{3} \quad (- \text{不合})$$

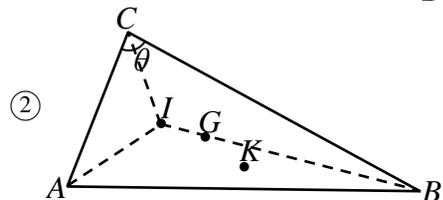
$$\textcircled{2} \quad x < -\frac{1}{2} \text{ 時, } x^2+1 = (-2x)^2 \Rightarrow 3x^2=1 \Rightarrow x = \pm \frac{\sqrt{3}}{3} \quad (+ \text{不合})$$

$$\text{由}\textcircled{1}、\textcircled{2}\text{得 } \frac{-4+\sqrt{7}}{3} \text{ 或 } \frac{-\sqrt{3}}{3}$$

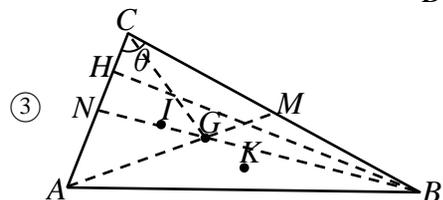
2. 在①、②、③中分別連接  $\overline{KA}$ 、 $\overline{KB}$ 、 $\overline{KC}$ 、 $\overline{IA}$ 、 $\overline{IB}$ 、 $\overline{IC}$ 、 $\overline{GA}$ 、 $\overline{GB}$ 、 $\overline{GC}$



$$\because K \text{ 為外心 } \therefore \angle KAC = \angle KCA, \angle KBC = \angle KCB \Rightarrow \angle AKB = 2\theta;$$



$$\because I \text{ 為內心 } \therefore \angle IAC = \frac{1}{2}\angle A, \angle IBC = \frac{1}{2}\angle B \Rightarrow \angle AIB = \theta + \frac{180^\circ - \theta}{2} = 90^\circ + \frac{\theta}{2};$$



$$\because G \text{ 為重心 } \therefore \text{延長 } \overline{GA} \text{ 至 } M \text{ 點、} \overline{GB} \text{ 至 } N \text{ 點, } M、N \text{ 分別為 } \overline{BC}、\overline{AC} \text{ 的中點}$$

$$\because \overline{CM} = \overline{AC} \therefore \angle GAC = \angle GMC \Rightarrow \angle GAC = \frac{180^\circ - \theta}{2}$$

$$\text{由 } B \text{ 點對 } \overline{AC} \text{ 作垂直線垂足為 } H \therefore \cos\theta = \frac{1}{8} \therefore \overline{CH} = \frac{1}{4} \text{ 又 } N \text{ 為 } \overline{AC} \text{ 中點 } \therefore \overline{CN} = \frac{1}{2} \Rightarrow \overline{NH} = \frac{1}{4} = \overline{CH}$$

$$\therefore \angle C = \angle BHC \Rightarrow \angle GBC = 180^\circ - 2\theta \Rightarrow \angle AGB = \theta + \frac{180^\circ - \theta}{2} + 180^\circ - 2\theta = 270^\circ - \frac{3\theta}{2}$$

$$\text{由}\textcircled{1}、\textcircled{2}、\textcircled{3}\text{得 } \angle AKB - \angle AIB + \angle AGB = 2\theta - (90^\circ + \frac{\theta}{2}) + (270^\circ - \frac{3\theta}{2}) = 180^\circ$$